Solving the TTC 2014 Movie Database Case with GrGen.NET

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The task of the Movie Database Case [2] is to identify all couples of actors who performed together in at least three movies. The challenge of the task is to do this fastly on a large database. We employ the general purpose graph rewrite system GRGEN.NET in developing and optimizing a solution.

1 What is GrGen.NET?

GRGEN.NET (www.grgen.net) is a programming language and development tool for graph structured data with pattern matching and rewriting at its core, which furthermore supports imperative and object-oriented programming, and features some traits of database query-result processing.

Founded on an rich data modeling language with multiple inheritance on node and edge types, it offers pattern-based rewrite rules of very high expressiveness, with subpatterns and nested alternative, iterated, negative, and independent patterns, as well as preset input parameters and output def variables yielded to. The rules are ochestrated with a concise control language, which offers constructs that are simplifying backtracking searches and state space enumeration.

Development is supported by graphical and stepwise debugging, as well as search plan explanation and profiling instrumentation for graph search steps – the former helps in first getting the programs correct, the latter in getting them fast thereafter. The tool was built for performance and scalability: its model data structures are designed for fast processing of typed graphs at modest memory consumption, while its optimizing compiler adapts the search process to the characteristics of the host graph at hand and removes overhead where it is not needed.

GRGEN.NET lifts the abstraction level of graph-representation based tasks to declarative and elegant pattern-based rules, that allow to program with structural recursion and structure directed transformation [4]. Employing GRGEN.NET one is able to achieve the highest combined speed of development and execution available for graph-based tasks.

2 Data transformation

The input supplied for the case is in XMI format, following the EMF modelling. This means esp. that in between Movie and Person nodes we find movies and persons references resp. edges, always a pair of them, one in the opposite direction of the other. These twins offer no addition information, they just allow to find the opposite node either way. This would be wasteful for GRGEN whose edges are always navigable in both directions.

So as a preparatory step we apply a model-to-text transformation, mapping the XMI models supplied to GRS files, the native serialization format of GRGEN.NET. The central rule <code>create_personToMovie</code> in <code>GrgenifyMovieDatabase.grg</code> emits a new edge command for a <code>personToMovie</code> edge upon visiting a pair of movies and <code>persons</code>. The transformation is applied on all input files with a shell script <code>GrgenifyMovieDatabase.sh</code> containing a call to the <code>GRSHELL</code> with a here-document specifying the workflow. In the following steps we then import the GRS files with the single edges.

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A further benefit from this model mapping are the cleaner names of the manually specified model, compared to the model that is auto-generated when the ecore/XMI is imported. The latter mechanism applies name mangling – which is needed to prevent name clashes of global edge types stemming from multiple equally named references that are contained in the type of their node, and for preventing collisions with GRGEN.NET keywords.

Removing the edges reduces memory consumption and improves cache utilization, the gains are considerable when working with millions of objects, esp. since GRGEN always implements directed, typed, multi-graphs with/for each of its entities. The memory consumption of a GRGEN.NET edge without attributes is 52 bytes using a 32bit CLR (9 pointers a 4 bytes, a 4 bytes flags field/bitvector, a 4 bytes unique id field, plus 8 bytes .NET object overhead), it is 96 bytes for a 64bit CLR (9 pointers a 8 bytes, a 4 bytes flags field, a 4 bytes unique id field, plus 16 bytes .NET object overhead). A node without attributes uses up 36 bytes, or 64 bytes on 64bit. Attributes only add the .NET overhead of their implementation type. The pointers stem from the system of ringlists the entities are contained in, which allows for fast pattern matching and rewriting (index-free adjacency).

3 Getting it right

As always, the first step is to get a correct solution specified in the cleanest way possible, and only later on to optimize it for performance as needed.

Main rule

The workhorse rule for finding all pairs of persons which played together in at least three movies is findCouples.

```
rule findCouples
{
  pn1:Person; pn2:Person;
  independent {
    pn1 -:personToMovie -> m1:Movie <-:personToMovie - pn2;
    pn1 -:personToMovie -> m2:Movie <-:personToMovie - pn2;
    pn1 -:personToMovie -> m3:Movie <-:personToMovie - pn2;
}

modify {
    c:Couple;
    c -:p1-> pn1;
    c -:p2-> pn2;

    exec(addCommonMoviesAndComputeAverageRanking(c, pn1, pn2));
}
} \ auto
```

Figure 1: findCouples rule

It specifies a pattern of two nodes pn1 and pn2 of type Person, and an independent pattern which asks for 3 nodes m1, m2, m3 of type Movie, each being the target of an edge of type personToMovie starting at pn1, and each being also the target of an edge starting at pn2. (Types are given after a colon,

the optional name of the entity may be given before the colon, for edges they are inscribed into the edge representation -->).

An independent pattern means for one that its content only needs to be searched and is not available for rewriting, and for the other that for each pn1 and pn2 in the graph, it is sufficient to find a single instance of the independent, even if the rule is requested to be executed on all available matches — without the independent we would get all the permutations of m1, m2, and m3 as different matches.

The rewriting is specified as nested pattern in modify mode, which means that newly declared entities will be created, and all entities from the matched pattern kept unless they are explicitly requested to be deleted. Here we create a new node c of type Couple, link it with edges of the types p1 and p2 to the nodes pn1 and pn2, and then execute the helper rule addCommonMoviesAndComputeAverageRanking on c, pn1, and pn2. The helper rule is used to create the commonMovies edges to all the movies both played in.

The auto keyword after the rule requests GRGEN.NET to generate a symmetry filter for matches from automorphic patterns. The pattern is automorphic because it may be matched with pn2 for pn1 and pn1 for pn2.

To get *all* pairs of persons which played together in at least three movies we execute the rule with all-bracketing from a graph rewrite sequence in the GRSHELL script MovieDatabase.grs, filtering away automorphic matches, with the syntax: exec [findCouples\auto].

The helper rule addCommonMoviesAndComputeAverageRanking eats all common movies with an iterated pattern which is matched as often as possible; in the rewrite part it links the Couple node to each such movie with a commonMovies edge. In the eval part used for attribute evaluation, the avgRating is computed as the sum of the ratings of the movies munched, divided by the count of the iterateds matched. The def var is used to define a variable whose content is computed after matching from the matched entites, the yield is used to assign to such variables, from nested patterns up to their containing patterns (normally variables are passed the other way round, from nesting to nested patterns, following the flow of matching).

The language constructs are explained in more detail in the extensive GRGEN.NET user manual [1].

Test data and Top Couples

The synthetic test set of task 1 is generated with the rules in MovieDatabaseCreation.grg. It is created in the very same way as in the Henshin specification given in the case description, just in textual GRGEN syntax, with a sequence definition createExample implemented with an integer range iteration for{i:int in [0:n-1]; createPositive(n) ;> createNegative(n)} and the rules createPositive and createNegative, applied in succession by the then-right operator.

You can check they are correctly generated with the graph viewer supplied with GRGEN.NET. In Figure 2 you see the smallest generated synthetic model, with actors colored yellow and movies colored blue.

The extension task 1 requires to compute the top couples according to the average rating of their common movies, and according to the number of common movies. It is solved with the rule couplesWithRating.

The rule matches a Couple and its linked Persons. Two def variables avgRating and num-Movies for the values of interest are created and filled with the average rating stored in the couple nodes, and the number of movies as computed from the size of the set of commonMovies edges outgoing from the couple node. We employ a yield block to assign the variables (bottom-up) after the (top-down) pattern matching completed. In the rewrite part specified in modify mode we just emit the

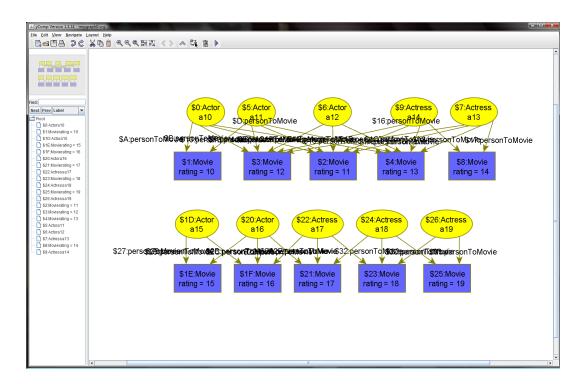


Figure 2: The smallest synthetic model in the graph viewer

values of interest. Furthermore, we request GRGEN to generate sorting code for the def pattern entities avgRating and numMovies with the declaration of the auto-generated matches accumulation filters orderDescendingBy<avgRating> and orderDescendingBy<numMovies>.

We use the rule twice, ordering the matches differently. The sequence [couplesWithRating\orderDescendingBy<avgRating>\keepFirst(15)] executed from GRSHELL asks for all matches of the rule couplesWithRating, then sorts them descendingly by the avgRating pattern variable, then throws away all but the first top 15 matches. The other value of interest is handled in exactly the same way.

You can inspect the found results with the debugger of GRGEN.NET. It highlights the matched pattern of the (currently executed) rule in the current graph, and displays the effects of the rewriting step before it leads to the follow-up graph. This only works with the full graph for the smaller autogenerated and the smallest IMDB-derived graph, though. The larger graphs are too costly to visualize (which simply poses a general problem of large graphs irrespective of the chosen viewer – besides you would get lost in a haystack). But we can configure the debugger with dump add graph exclude to show only the matched entities, plus the one-step connected elements as context. This way we can still inspect what GRGEN found. In Figure 4 can you see a visualization of the match with most connections, in layout Circular (take a look at MovieDatabaseLayout.grs to find out about the visualization configuration applied to reach this). We achieved this by executing the sequence

 $\$ [couplesWithRating\orderDescendingBy<numMovies>\removeFirst(0)\keepFirst(1)] in debug mode. First all matches are sought, then they are ordered descendingly alongside the number of common movies, then the first 0 matches (i.e. nothing) are removed – increase this value to k to inspect the kth match from the matches found. Afterwards, all but the first 1 remaining match is removed – increase this value to k to inspect the first k remaining matches at once. The k chooses one match from

Figure 3: couplesWithRating rule

all matches found that remain after sorting and filtering, the % allows you to cycle through the matches in the debugger.

Cliques

The other extension tasks asking to find cliques of actors are solved with manually coded rules for the sizes 3,4, and 5 as a straightforward generalization of the couples-based task. GRGEN.NET does not support built-in higher-order or meta-programming (and it is unlikely it will do so in the future, because in our eyes its cost-benefit ratio is too bad: it is seldom needed in practice, yields badly understandable code, and is difficult to implement in a compiled approach).

4 Getting it fast

The most important point to understand when optimizing for speed is that the expensive task is the search carried out during pattern matching.

Searching is carried out with a backtracking algorithm following a search plan in a fixed order, binding one pattern element after another to a graph element, checking if it fits to the already bound parts. If it does fit search continues trying to bind the next pattern element (or succeeds building the match object from all the elements bound if the last check succeeds), if it does not fit search continues with the next graph element; if all graph element candidates for this pattern element are exhausted, search backtracks to the previous decision point and continues there with the next element.

Typically, the first pattern element is determined with a lookup operation reaching into the graph, binding the pattern element to a graph element of the required type (the less elements of that type exists, the better); then neighbouring elements are traversed following the graph structure (the less neighbouring elements exists, the better), until a match of the entire pattern is found.

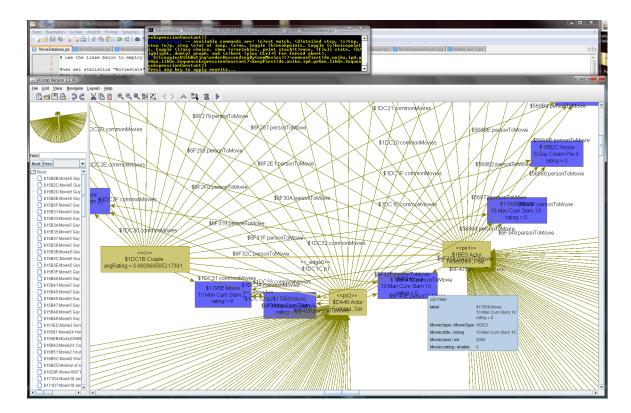


Figure 4: Debugging the top match from the 10000 movies file

Initial and Host-Graph-Adapted Search Plans

For each pattern, GRGEN.NET creates a static un-optimized search plan. You can inspect it for each rule with the custom actions explain command. For findCouples it delivers the output displayed in Figure 5.

The graph is entered with a lookup of a personToMovie edge, then the candidates for the source node pn1 and the target node m1 are extracted from it. Afterwards the other personToMovie edge is matched in reverse from the movie to pn2. Several elements in the independent are handed in as already matched presets from the outer pattern, then the personToMovie edges are taken from pn1 to the movies m2 and m3, and finally the personToMovie edges from pn2 are matched, with an implicit check that the target movie is the same as the one already matched.

We see here an automatically applied optimization, the movie m1 was inlined from the independent pattern to its containing pattern. Without this optimization, pn1 and pn2 would have to be enumerated in the main pattern *unconnected*, resulting in the unfolding of the cartesian product of all Person nodes, before handing it in to the matcher of the nested independent pattern to purge the actors without a connecting movie. This is a crucial optimization that saves us from the execution time explosion of the O(n*n) cartesian product.

We can do better with search plans adapted to the host graph, towards this end we issue a custom graph analyze followed by a custom actions gen_searchplan command. The first computes some information about the multiplicities of the types and esp. the multiplicities of the connections in between the types from the host graph, the latter re-generates the matchers at runtime based on those graph characteristics gathered, replacing the auto-generated ones. For the small IMDB graphs, this

```
static search plans
findCouples:
    lookup -_edge0_inlined_idpt_0:personToMovie-> in graph
    from <-_edge0_inlined_idpt_0- get source pn1:Person</pre>
    from -_edge0_inlined_idpt_0-> get target m1_inlined_idpt_0:Movie
    from m1_inlined_idpt_0 incoming <-_edge1_inlined_idpt_0:personToMovie-
    from <-_edge1_inlined_idpt_0- get source pn2:Person</pre>
    independent {
        (preset: pn1)
        (preset: m1 after independent inlining)
        (preset: pn2)
        (preset: _edge0 after independent inlining)
        (preset: _edge1 after independent inlining)
        from pn1 outgoing -_edge2:personToMovie->
        from -_edge2-> get target m2:Movie
        from pn1 outgoing -_edge4:personToMovie->
        from -_edge4-> get target m3:Movie
        from pn2 outgoing -_edge3:personToMovie->
        from pn2 outgoing -_edge5:personToMovie->
    }
```

Figure 5: Initial search plan

results in a lookup of m1, extending on the one side to pn1, and on the other to pn2. For the large IMDB graph this results in a lookup of pn1, extending to m1, then extending to pn2; this leads to a small runtime improvement, esp. together with a later attribute condition.

Profiling and Pruning

Things still are getting really slow with increasing graph sizes for the IMDB graphs. We must go into greater details – let's start profiling. We do so with the new set profile on command at the begin of the shell file before the graph is created, and a show profile findCouples command thereafter. We see massive increases in the number of search steps with the graph sizes, hinting at an O(n*n) algorithm. Inspecting the top connected matches visually and aligning them with the search plan we understand why: the pattern matcher has to follow all edges from pn1 on and all edges from pn2 on, to find and bind the common movies, giving us an O(n*n) iteration (with n being the number of adjacent movies). The iteration is not much of an issue for actors with few connections (i.e. sparse graphs), but gets very expensive for massively connected actors. Here we simply have to revert to a non-pattern based approach to compute the set of common nodes. You see it displayed in Figure 6 with an intersection of two sets, which is O(n) because the hash set lookup employed by the intersection operator & is O(1).

We applied a further optimization that prunes the search space earlier. Instead of finding and materializing the automorphic matches, just to remove them later on with an automorphic matches filter, we reject them straight ahead. To this end, we require that the unique id of the actor bound to pn1 is smaller than the unique id of the actor bound to pn2. Unique ids are unique integer numbers assigned by GRGEN.NET after they have been requested with a node edge unique declaration in the model (they

```
rule findCouplesIntermediateOpt1
{
   pn1:Person; pn2:Person;
   independent {
      pn1 -:personToMovie -> m1:Movie <-:personToMovie - pn2;
      if{ (adjacentOutgoing(pn1, personToMovie) &
            adjacentOutgoing(pn2, personToMovie)).size() >= 3; }
}
if{ uniqueof(pn1) < uniqueof(pn2); }

modify {
      c:Couple;
      c -:p1-> pn1;
      c -:p2-> pn2;

      exec( [addCommonMoviesIntermediateOpt(c, pn1, pn2)] );
}
```

Figure 6: 1st optmization step: findCouplesIntermediateOpt1

render deletion a bit more expensive, which is why they are not active by default).

Furthermore, we optimize addCommonMoviesAndComputeAverageRanking. Instead of iterating all common movies from within one rule application, computing the averages from the iterated patterns, do we only add the common movies in this step without averaging (more is not requested), with an all-bracketed application of addCommonMovies which saves us the overhead of the graph parser employed in matching iterated, alternative, and subpatterns. Those costs are not huge, but they can be felt.

Further Pruning and Strength Reduction

The set intersection already helped regarding the big-O, we still can do better: only 3 common movies are needed, so we replace it with a dedicated algorithm that iterates one set and checks whether the movies are available in the other, leaving early after 3 such movies were found. In excess, this algorithm only needs to allocate and fill one single set, which saves us considerable overhead.

The helper function atLeastThreeCommonMoviesIntermediateOpt2 displayed in Figure 12 takes two Persons pn1 and pn2 as arguments and returns true if they are linked by at least three common movies, and returns false otherwise. For each movie, it checks with a hash set lookup in O(1) whether it is common to the persons.

We even use an adaptive helper function here; we first find out what is the node with the smaller amount of adjacent Movie nodes by counting them, before we build the hash set from that smaller neighbourhood. The extra effort of counting pays off, hash set allocating and building is expensive (which is why we don't use hash sets in implementing the pattern matchers, in most cases they lead to a considerable slowdown compared to the nested loops employed there).

We prune the search space still further by exploiting the "long tail", the fact that there are many actors existing that performed in less than 3 movies (in strong contrast to the massively connected actors that cause the major pattern matching issues). So we check in the resulting findCouplesIntermediateOpt2 given in Figure 13 for actors with at least 3 adjacent movies.

Finally – for this optimization step – we introduce a helper procedure addCommonMoviesInter-

mediateOpt shown in Figure 14 that employs the same set based algorithm for adding commonMovies edges to *all* the common nodes.

Index usage and Parallelization

In the previous section we introduced quite some adjacent node counting (equalling incident edge counting). We can do better here with an index. GRGEN supports attribute indices for quick access to graph elements based on their attribute values, and incidence count indices for quick access to nodes based on their number of incident edges, maintained in the background by the engine. Here we declare an incident edge index countPersonToMovie in the model file, and use it in the rules with countPersonToMovie[element] to get O(1) access to the needed counts instead of having to iterate the list of incident edges, counting them.

You see a further minor optimization in the resulting rule with application of hom-declarations, they allow the pattern elements to match the same graph elements. The default is isomorophy, which needs to be checked. This is done efficiently, but here we want to squeeze out as much as possible, so we remove the checks – we can do so because the unique id checking already ensures that the pattern elements are not matched to the same host graph elements.

Until now we were concerned with minimizing the amount of work to be carried out for the specification. But with ourdays multicore machines there is an alternative strategy available: maximize the amount of workers thrown on the work. Applying both as possible will of course yield the best results.

Parallelization brings locking and work distribution overhead with it, so it is not for free performance-wise. For tasks with only little search work to be carried out it makes things run *slower*. But a task with an expensive search like ours – remember the huge number of search steps shown by profiling – benefits considerably from it.

You apply parallelization in GRGEN.NET with a parallelize annotation at a rule, specifying the maximum amount of worker threads to use. Only the matcher is parallelized, and only the first search operation which causes a splitting in the search space, i.e. a lookup that splits work alongside all elements of the type requested, or an extension with a edge that splits work alongside all incident edges. (The full transformation including rewriting won't be parallelized as it was done in the prototype employed in [5] because of the in our eyes too bad cost-benefit-ratio: the task where most benefits can be gained is searching for a match, a parallelization of rewriting would render the programming model to the user much more complicated, and would require graph partitioning (locking would eat up all performance gains), which is a difficult problem on its own, and esp. so for a general-purpose tool that must work for many different graph representations).

The final optimized rule is shown in Figure 7, and its helper functions and procedures in Figure 10 and Figure 11; its search plan is listed in Figure 8.

5 Calling from API and Performance Results

The task description asks for a standalone command line version for benchmarking. We supply a C#-Program that employs the GRGEN.NET API towards this end, which can be found in MovieDatabaseBenchmarker.cs. It expects as first parameter the name of the rule to apply (findCouplesOpt, findCliquesOft), findCliqueOftOpt), and as second parameter either the graph to import (e.g. imdb-0005000-50176.movies.xmi.grs), or the number of iterations to use in generating the synthetic test graph. An additional sequence may be given in quotes, intended for emitting the sorted

```
rule findCouplesOpt[parallelize=16]
  pn1:Person; pn2:Person;
  hom(pn1,pn2);
  independent {
    pn1 -p2m1:personToMovie-> m1:Movie <-p2m2:personToMovie- pn2;</pre>
    hom(pn1,pn2); hom(p2m1,p2m2);
    if{ atLeastThreeCommonMovies(pn1, pn2); }
  if{ uniqueof(pn1) < uniqueof(pn2); }</pre>
  if{ countPersonToMovie[pn1] >= 3; }
  if{ countPersonToMovie[pn2] >= 3; }
  modify {
    c:Couple;
    c -: p1-> pn1;
    c -: p2 -> pn2;
    eval { addCommonMovies(pn1, pn2, c); }
 }
}
```

Figure 7: findCouplesOpt rule

results.

The standalone version contains a further optimization that can be only applied on API level. After importing the IMDB graphs, it reduces the named graphs to unnamed graphs, throwing away the name information. The GRSHELL always employs named graphs, they supply the names that can be seens in the debugger, and the names that are written in grs-serialization and read in again in de-serialization. The name information consists of a hash map from names to graph elements, and one hash map from graph elements to names. That sounds rather innocent, but requires about as much storage space as the plain GRGEN-graph, which accumulates to gigabytes for the large test graphs. Throwing this information away reduces the maximum memory requirements considerably, saves the engine from name maintenance on inserts, and allows for better cache utilization (depending on the allocation pattern of the .NET runtime).

6 Conclusion

We first specified a clean and simple solution of the movie database task in the GRGEN languages, then we optimized it for performance. The tool supported us in validating the solution with its graphical debugger, and in optimizing it with its search plan explanation and profiling instrumentation for search step counting.

GRGEN.NET search planning can be compared to searching straw stars on a freshly harvested field, by looking at the places where the ground is only slightly covered, only reaching into the haystacks when they can't be circumvented at all. A pattern matcher is generated based on the assumption that search planning worked well in circumventing those haystacks. Here the task consists solely of diving within a hay stack, a particularily large and interwoven one. So we had to replace to a considerable degree the elegant and declarative pattern matcher with its loop-nesting based implementation (being well adapted to sparse graphs), by imperative hash set based helper functions – highlighting the availability of the

```
static search plans from Moviestats
findCouplesOpt:
    parallelized lookup pn1:Person in graph
    if { depending on findCouplesOpt_node_pn1, }
    from pn1 outgoing -p2m1_inlined_idpt_0:personToMovie->
    from -p2m1_inlined_idpt_0-> get target m1_inlined_idpt_0:Movie
    from m1_inlined_idpt_0 incoming <-p2m2_inlined_idpt_0:personToMovie-
    from <-p2m2_inlined_idpt_0- get source pn2:Person</pre>
    if { depending on findCouplesOpt_node_pn2, }
    if { depending on findCouplesOpt_node_pn1,findCouplesOpt_node_pn2, }
    independent {
        (preset: pn1)
        (preset: m1 after independent inlining)
        (preset: pn2)
        if { depending on findCouplesOpt_node_pn1,findCouplesOpt_node_pn2, }
        (preset: p2m1 after independent inlining)
        (preset: p2m2 after independent inlining)
    }
```

Figure 8: Final search plan

right language constructs for about any task built on a graph-based representation.

On a quad core processor, the synthetic benchmark graph was processed in about 100sec for the couples, the 3 cliques in about 55sec, the 4 cliques in about 28sec, and the 5 cliques in about 56sec. The entire IMDB was searched for all couples performing in at least three movies in about 420sec, the found couples were linked to all of their common movies in about another 200sec. The cliques show a stunning computation time explosion, from 5 secs for cliques of 3, over something less than 2 minutes for cliques of 4, to about 2 and a half hours for cliques of 5, already on the smallest IMDB graph.

References

- [1] Jakob Blomer, Rubino Geiß & Edgar Jakumeit (2014): *The GrGen.NET User Manual.* http://www.grgen.net.
- [2] Tassilo Horn, Christian Krause & Matthias Tichy (2014): The TTC 2014 Movie Database Case.
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- [5] Jochen Schimmel, Tom Gelhausen & Christoph A. Schaefer (2009): Gene Expression with General Purpose Graph Rewriting Systems. In: Proceedings of the 8th GT-VMT Workshop, Electronic Communications of the EASST 18. Available at journal.ub.tu-berlin.de/eceasst/article/view/276/259.

A Code Listings

```
rule addCommonMoviesAndComputeAverageRanking(c:Couple, pn1:Person, pn2:Person)
{
  iterated it {
    pn1 -:personToMovie-> m:Movie <-:personToMovie- pn2;

  modify {
        c -:commonMovies-> m;
        eval { yield sum = sum + m.rating; }
    }
}

modify {
    def var sum:double = 0.0;
    eval { c.avgRating = sum / count(it); }
}
```

Figure 9: addCommonMoviesAndComputeAverageRanking rule

```
function atLeastThreeCommonMovies(pn1:Person, pn2:Person) : boolean
  if(countPersonToMovie[pn1] <= countPersonToMovie[pn2])</pre>
    def var common: int = 0;
    def ref movies:set<Node> = adjacentOutgoing(pn1, personToMovie);
    for(movie:Node in adjacentOutgoing(pn2, personToMovie))
      if(movie in movies) {
        common = common + 1;
        if(common >= 3) {
          return(true);
     }
    }
 }
  else
  {
    def var common: int = 0;
    def ref movies:set<Node> = adjacentOutgoing(pn2, personToMovie);
    for(movie:Node in adjacentOutgoing(pn1, personToMovie))
      if(movie in movies) {
        common = common + 1;
        if(common >= 3) {
          return(true);
     }
   }
 }
  return(false);
```

Figure 10: atLeastThreeCommonMovies helper function

```
procedure addCommonMovies(pn1:Person, pn2:Person, c:Couple)
  if(countPersonToMovie[pn1] >= countPersonToMovie[pn2])
    def ref movies:set<Node> = adjacentOutgoing(pn2, personToMovie);
   for(movie:Node in adjacentOutgoing(pn1, personToMovie))
      if(movie in movies) {
        add(commonMovies, c, movie);
    }
 }
  else
    def ref movies:set<Node> = adjacentOutgoing(pn1, personToMovie);
    for(movie:Node in adjacentOutgoing(pn2, personToMovie))
      if(movie in movies) {
        add(commonMovies, c, movie);
   }
 }
  return;
```

Figure 11: addCommonMovies helper procedure

```
function \ at Least Three Common Movies Intermediate Opt 2 (pn 1: Person, pn 2: Person) : \textit{boolean}
  if(countAdjacentOutgoing(pn1, personToMovie) <= countAdjacentOutgoing(pn2, personToMovie))</pre>
    def var common: int = 0;
    def ref movies:set<Node> = adjacentOutgoing(pn1, personToMovie);
    for(movie:Node in adjacentOutgoing(pn2, personToMovie))
      if(movie in movies) {
        common = common + 1;
        if(common >= 3) {
          return(true);
      }
    }
 }
  else
  {
    def var common: int = 0;
    def ref movies:set<Node> = adjacentOutgoing(pn2, personToMovie);
    for(movie:Node in adjacentOutgoing(pn1, personToMovie))
      if(movie in movies) {
        common = common + 1;
        if(common >= 3) {
          return(true);
      }
    }
 }
  return(false);
```

Figure 12: atLeastThreeCommonMoviesIntermediateOpt2 helper function

```
rule findCouplesIntermediateOpt2
{
   pn1:Person; pn2:Person;
   independent {
      pn1 -:personToMovie-> m1:Movie <-:personToMovie- pn2;
      if{ atLeastThreeCommonMoviesIntermediateOpt2(pn1, pn2); }
}

if{ uniqueof(pn1) < uniqueof(pn2); }

if{ countAdjacentOutgoing(pn1, personToMovie) >= 3; }

if{ countAdjacentOutgoing(pn2, personToMovie) >= 3; }

modify {
      c:Couple;
      c -:p1-> pn1;
      c -:p2-> pn2;

      exec( [addCommonMoviesIntermediateOpt(c, pn1, pn2)] );
}
```

Figure 13: findCouplesIntermediateOpt2 helper function

```
rule addCommonMoviesIntermediateOpt(c:Couple, pn1:Person, pn2:Person)
{
   pn1 -:personToMovie-> m:Movie <-:personToMovie- pn2;

   modify {
      c -:commonMovies-> m;
   }
}
```

Figure 14: addCommonMoviesIntermediateOpt helper procedure